Infrared divergence of pure Einstein gravity contributions to cosmological density power spectrum

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KIAS

Workshop on DM, LHC and Cosmology
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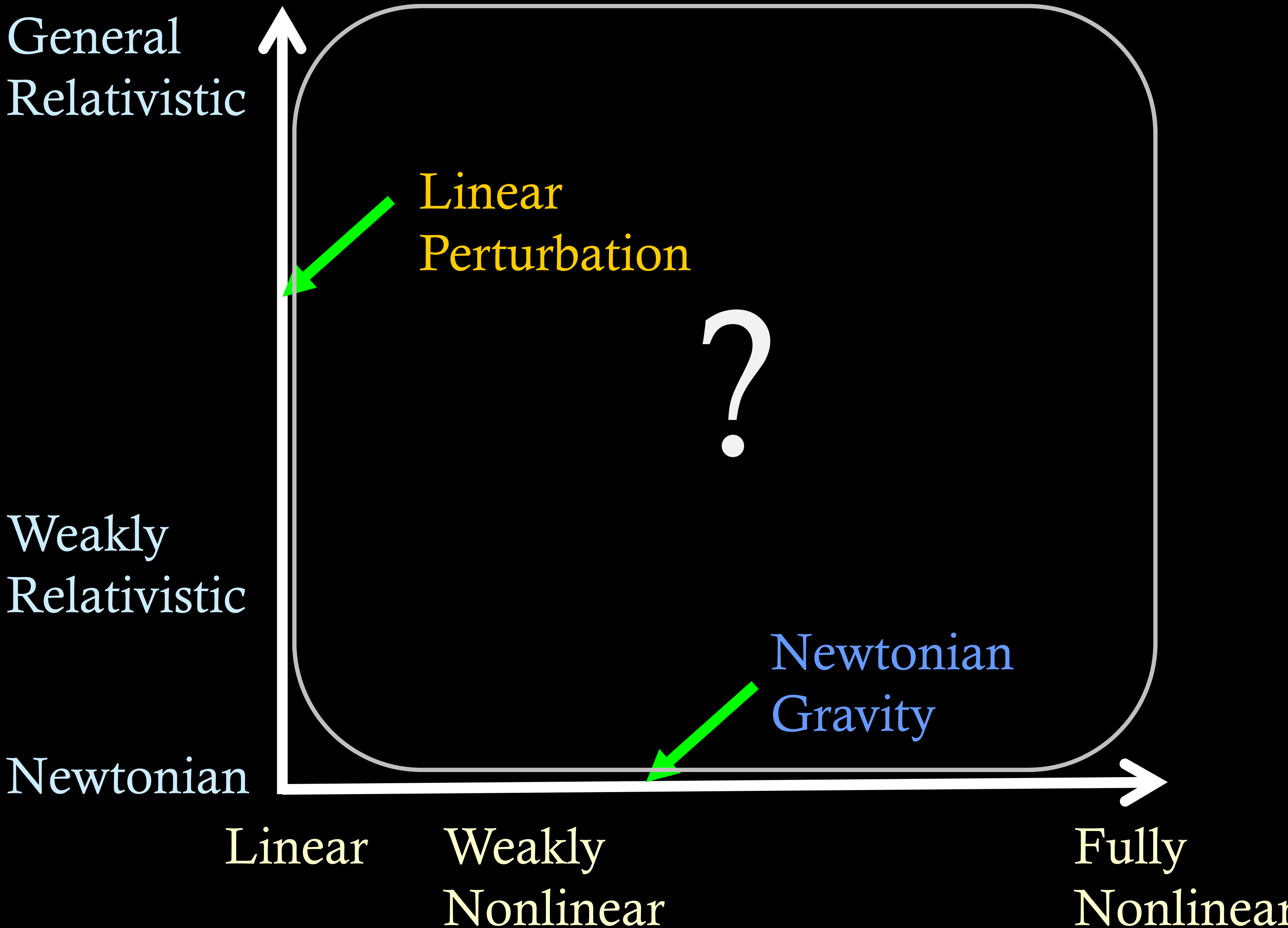
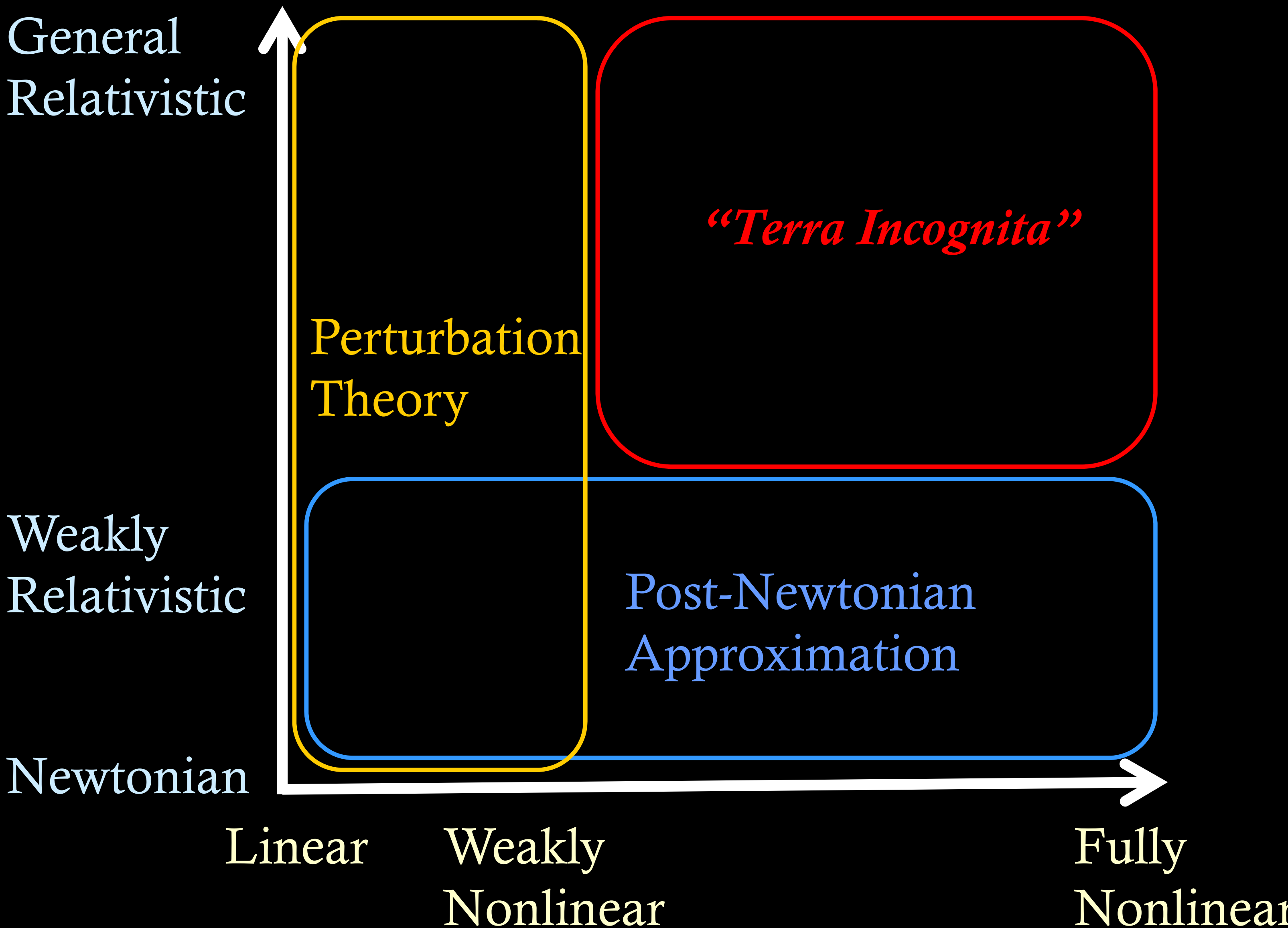
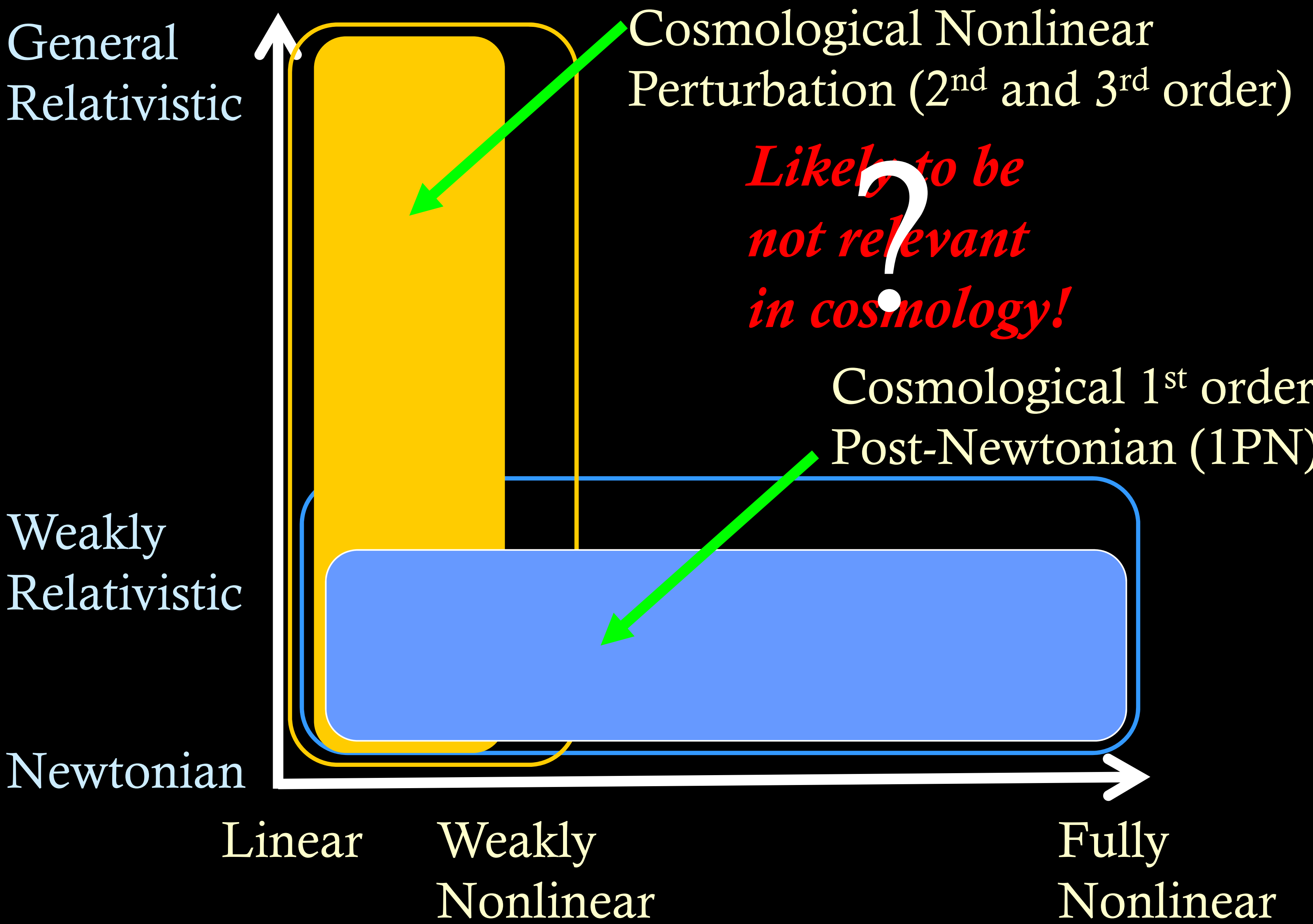
Studies of Large-scale Structure



Perturbation Theory *vs.* Post-Newtonian



Cosmology and Large-Scale Structure



Perturbation method:

- Perturbation expansion.
- All perturbation variables are small.
- Weakly nonlinear.
- Strong gravity; fully relativistic!
- Valid in all scales!

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time.
- Provide GR correction terms in the Newtonian equations of motion.
- Expansion in $\frac{\delta\Phi}{c^2} \sim \frac{GM}{Rc^2} \sim \frac{v^2}{c^2} << 1$
- Fully nonlinear!
- No strong gravity situation; weakly relativistic.
- Valid far inside horizon

Relativistic/Newtonian correspondence:

Background order:

Spatial curvature / Total energy

Cosmological constant

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\varrho - \frac{\text{const}}{a^2} + \frac{\Lambda c^2}{3},$$

Friedmann (1922)/Milne and McCrea (1934)

Linear perturbation:

Density

Density perturbation/Density

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\varrho\delta = 0,$$

Lifshitz (1946)/Bonnor (1957)

Linear order: Lifshitz (1946)/Bonnor (1957) (comoving-synchronous gauge)

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = 0,$$

Second order: Peebles (1980)/Noh-Hwang (2004) (K=0, comoving gauge)

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}),$$

Third order: Hwang-Noh (2005) (K=0, comoving gauge)

Curvature perturbation in the comoving gauge ~10^{-5}

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u})$$

$$+ \frac{1}{a^2}\frac{\partial}{\partial t}\left\{a\left[2\varphi\mathbf{u} - \nabla(\Delta^{-1}X)\right]\cdot\nabla\delta\right\} - \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right]$$

$$+ \frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla(\nabla\cdot\mathbf{u}) + \frac{\Delta}{a^2}\left[\mathbf{u}\cdot\nabla(\Delta^{-1}X)\right] - \frac{1}{a^2}\mathbf{u}\cdot\nabla X - \frac{2}{3a^2}X\nabla\cdot\mathbf{u},$$

$$X \equiv 2\varphi\nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\varphi + \frac{3}{2}\Delta^{-1}\nabla\cdot\left[\mathbf{u}\cdot\nabla(\nabla\varphi) + \mathbf{u}\Delta\varphi\right].$$

Pure General Relativistic corrections

Physical Review D, 72, 044012 (2005).

General relativistic contributions to second-order power spectrum:

$$\delta \equiv \delta_1 + \delta_2 + \delta_3 + \cdots$$

$$P \equiv \langle |\delta|^2 \rangle = \langle |\delta_1|^2 \rangle + \langle |\delta_2|^2 \rangle + 2\langle \mathrm{Re}(\delta_1^* \delta_3) \rangle + \cdots$$

$$\equiv P_{11} + P_{22} + P_{13} + \cdots$$

Pure General Relativistic contribution!

$$\delta_3 = \delta_{3,Newton} + \delta_{3,Einstein}$$

$$P_{13} = P_{13,Newton} + P_{13,Einstein}$$

Relativistic / Newtonian

Density power spectrum to second-order:

K=0 =Λ:

$$|\delta(\mathbf{k},t)|^2 = |\delta_1(\mathbf{k},t)|^2 + \frac{1}{(2\pi)^3}\int d^3k' \Bigg\{ \frac{2}{14^2}|\delta_1(\mathbf{k}',t)|^2|\delta_1(\mathbf{k}-\mathbf{k}',t)|^2 J^2(\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}')$$
$$+|\delta_1(\mathbf{k},t)|^2\Bigg[|\delta_1(\mathbf{k}',t)|^2\bigg(\frac{2}{63}F(\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}')L(\mathbf{k}-\mathbf{k}',\mathbf{k},-\mathbf{k}')$$
$$+\frac{1}{18}H(\mathbf{k},\mathbf{k}')J(\mathbf{k}-\mathbf{k}',\mathbf{k},-\mathbf{k}') + \frac{1}{18}H(\mathbf{k},\mathbf{k}-\mathbf{k}')L(\mathbf{k}-\mathbf{k}',\mathbf{k},-\mathbf{k}')\bigg) + C^+\Bigg]\Bigg\}$$

Newtonian

$P_{22} + P_{13,\mathrm{Newton}}$

$$+\frac{10}{21}\left(\frac{\ell}{\ell_H}\right)^2|\delta_1(\mathbf{k},t)|^2\frac{1}{(2\pi)^3}\int d^3k'\Bigg\{|\delta_1(\mathbf{k}',t)|^2\left(\frac{13}{3}+4\frac{k^2}{k'^2}+7\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}-12\frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k'^4}+14\frac{k^2}{k'^2}\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}\right)$$
$$+\Bigg[|\delta_1(\mathbf{k}',t)|^2 M(\mathbf{k}-\mathbf{k}',\mathbf{k},-\mathbf{k}')\frac{k^2}{|\mathbf{k}-\mathbf{k}'|^2}\left(1-3\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}+\frac{15}{2}\frac{\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|^2}+3\frac{k^2}{k'^2}\frac{\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|^2}\right)+C^+\Bigg]\Bigg\},$$

Pure General Relativistic corrections $P_{13,\mathrm{Einstein}}$

$$M(\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}') \equiv \frac{k^2}{k'^2}+\frac{k^2}{|\mathbf{k}-\mathbf{k}'|^2}+\frac{3}{4}\frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}+\frac{3}{4}\frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|^2}-\frac{1}{4}\frac{k^2}{k'^2}\frac{\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')}{|\mathbf{k}-\mathbf{k}'|^2}.$$

$$\ell/\ell_H \equiv \dot{a}/(kc)$$

Physical Review D, 77,123533 (2008)

P_{11} P_{22}

$$\langle|\delta(k,t)|^2\rangle = \langle|\delta_1(k,t)|^2\rangle + \frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty dr\,|\delta_1(kr,t)|^2 \int_{-1}^{1} dx \left|\delta_1\left(k\sqrt{1+r^2-2rx},t\right)\right|^2 \frac{\left(3r+7x-10rx^2\right)^2}{\left(1+r^2-2rx\right)^2}$$
$$+\frac{1}{252}\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-42r^4+100r^2-158+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(7r^2+2\right)\ln\left|\frac{1+r}{1-r}\right|\right]$$
$$+\frac{10}{21}\left(\frac{\ell}{\ell_H}\right)^2\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-\frac{41}{6}r^2-21-\frac{45}{4}\frac{1}{r^2}\right.$$
$$\left.+\frac{9}{8}\frac{1}{(1+r)^2(1-r)^2}\left(5r^6-13r^4+9r^2+1-\frac{2}{r^2}\right)+\frac{3}{16}\left(43r^2+46r-\frac{53}{r}-\frac{36}{r^3}\right)\ln\left|\frac{1-r}{1+r}\right|\right]$$
$$\equiv P_{11}+P_{22}+P_{13,\mathrm{Newton}}+P_{13,\mathrm{Einstein}},$$

$P_{13,\mathrm{Newton}}$

$P_{13,\mathrm{Einstein}}$

where $r \equiv k'/k$ and $x \equiv (\mathbf{k}\cdot\mathbf{k}')/(kk')$; $\ell/\ell_H \equiv \dot{a}/(kc)$

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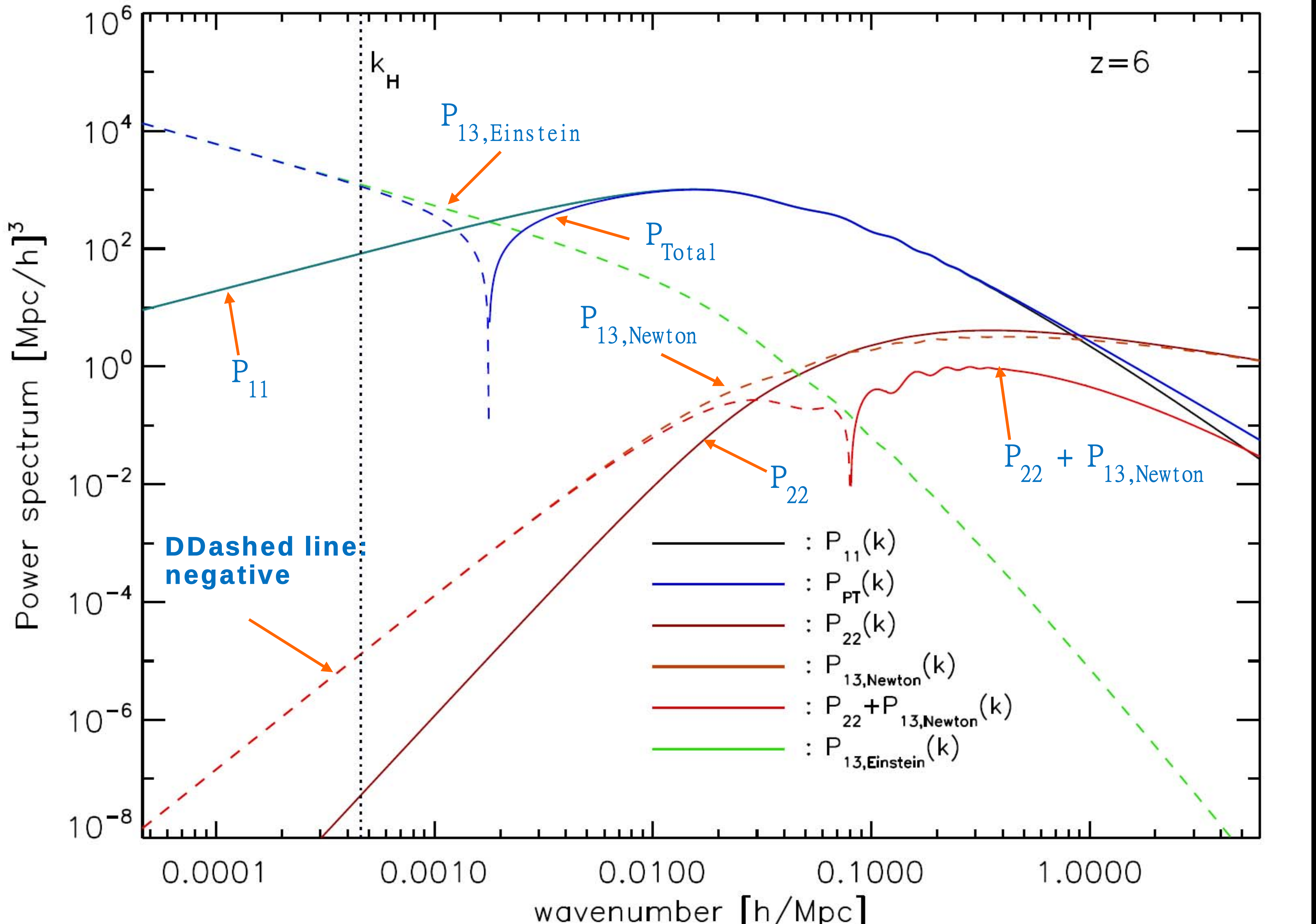

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Small-scale limit (k→∞, r→0):

Leading order, Newton (Vishniac 1983)

P_{22}

$$
\begin{aligned}
\langle|\delta(k,t)|^2\rangle =& \langle|\delta_1(k,t)|^2\rangle + \frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty dr\,|\delta_1(kr,t)|^2 \int_{-1}^{1} dx\,\left|\delta_1\left(k\sqrt{1+r^2-2rx},t\right)\right|^2 \frac{\left(3r+7x-10rx^2\right)^2}{\left(1+r^2-2rx\right)^2} \\
&+\frac{1}{252}\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-42r^4+100r^2-158+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(7r^2+2\right)\ln\left|\frac{1+r}{1-r}\right|\right] \\
&+\frac{10}{21}\left(\frac{\ell}{\ell_H}\right)^2\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-\frac{41}{6}r^2-21-\frac{45}{4}\frac{1}{r^2}\right. \\
&\left.+\frac{9}{8}\frac{1}{(1+r)^2(1-r)^2}\left(5r^6-13r^4+9r^2+1-\frac{2}{r^2}\right)+\frac{3}{16}\left(43r^2+46r-\frac{53}{r}-\frac{36}{r^3}\right)\ln\left|\frac{1-r}{1+r}\right|\right] \\
\equiv& P_{11}+P_{22}+P_{13,\mathrm{Newton}}+P_{13,\mathrm{Einstein}},
\end{aligned}
$$

$P_{13,\mathrm{Newton}}$

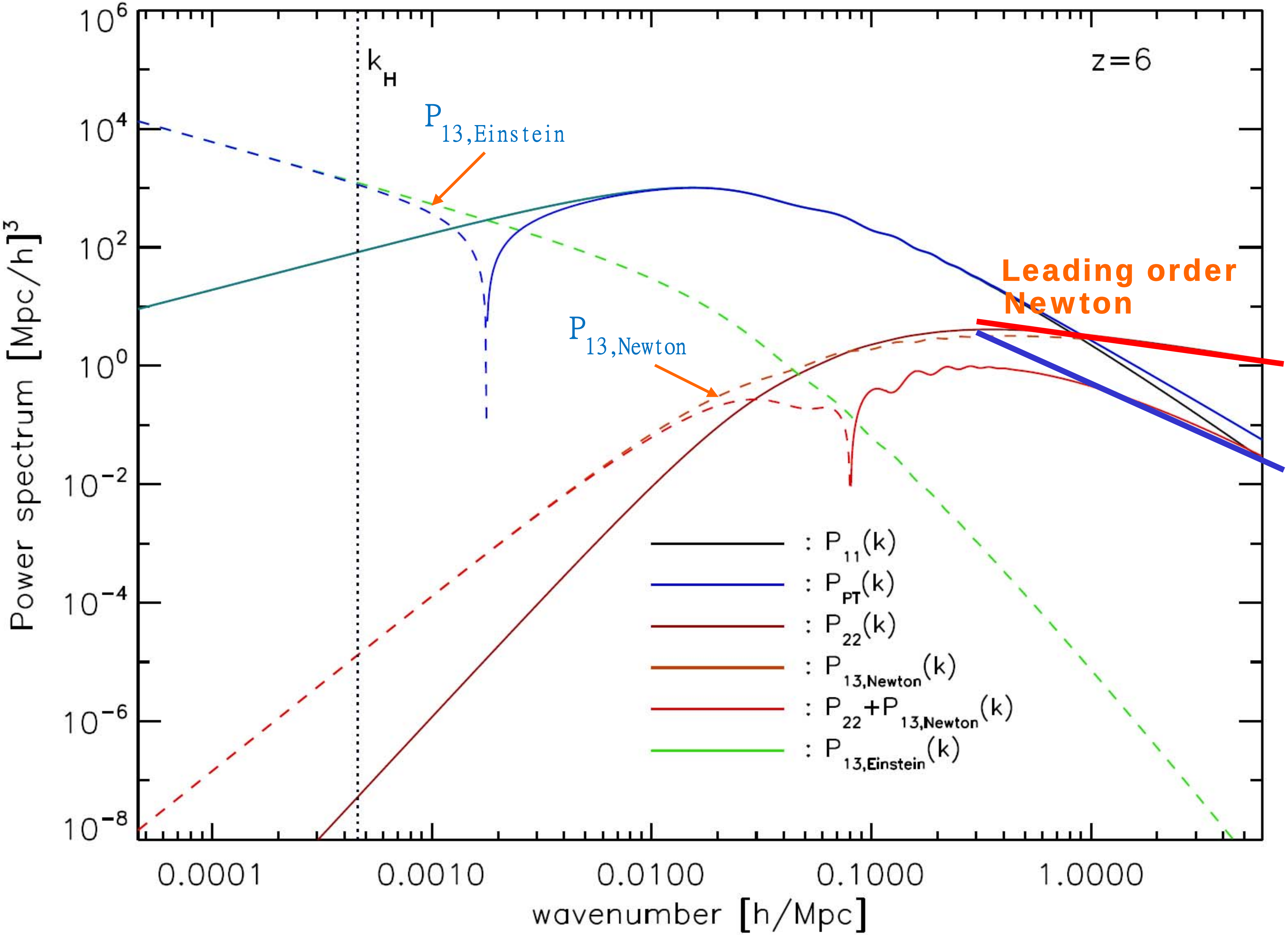
$P_{13,\mathrm{Einstein}}$

Leading order, Einstein

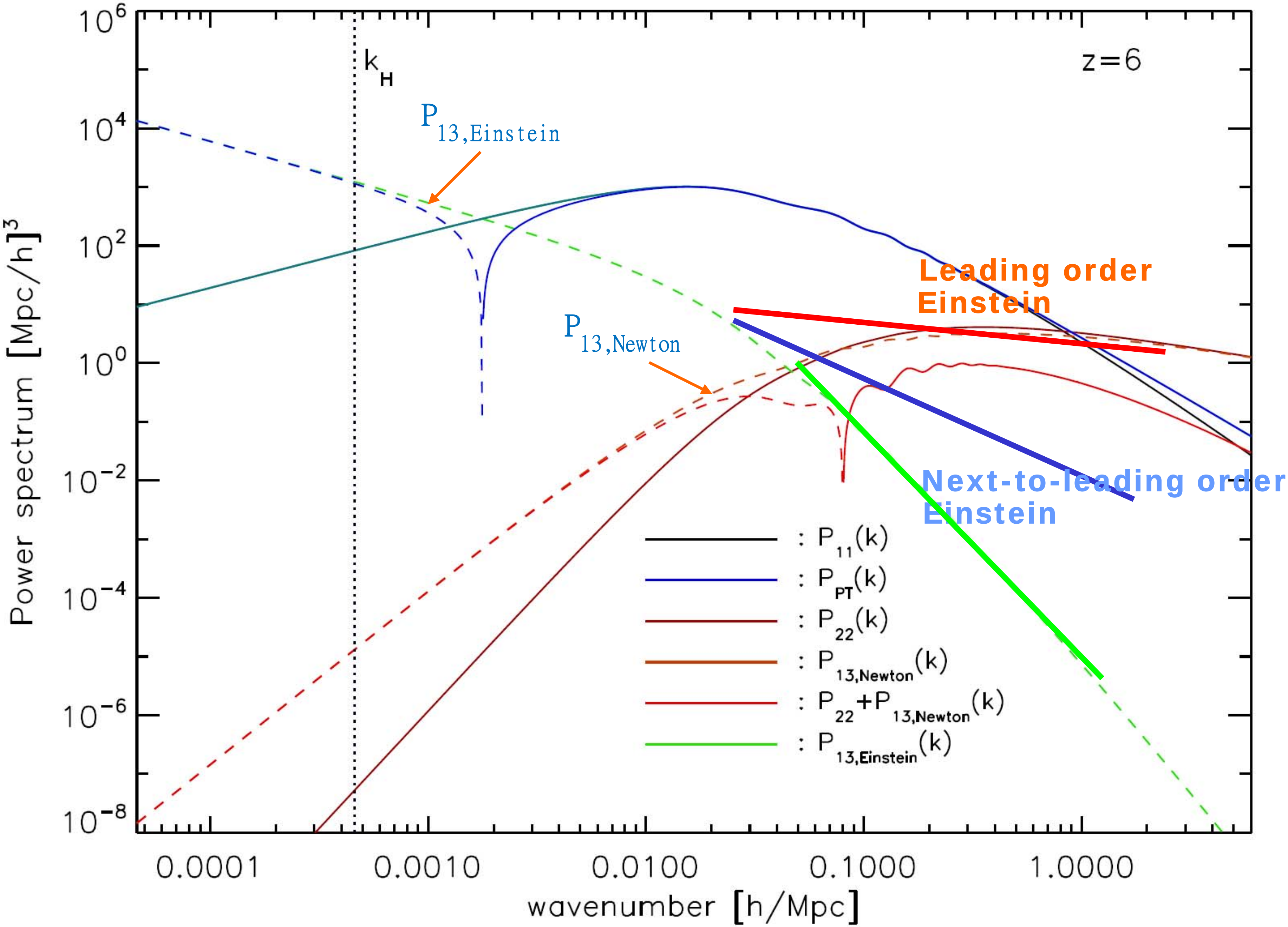
Next-to-leading order, Einstein

In the small-scale the Einstein's gravity contribution is still negligibly small.

← Due to cancellation of the leading-order and next-to-leading order terms in $P_{13,\mathrm{Einstein}}$, despite a cancellation between P_{22} in $P_{13,\mathrm{Newton}}$.



z=6
k_H
P_13,Einstein
P_13,Newton
Leading order Newton
: P_11(k)
: P_PT(k)
: P_22(k)
: P_13,Newton(k)
: P_22+P_13,Newton(k)
: P_13,Einstein(k)
Power spectrum [Mpc/h]^3
wavenumber [h/Mpc]
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-6
10^-8
0.0001
0.0010
0.0100
0.1000
1.0000



z=6
k_H
$P_{13,Einstein}$
$P_{13,Newton}$
Leading order
Einstein
Next-to-leading order
Einstein
: $P_{11}(k)$
: $P_{PT}(k)$
: $P_{22}(k)$
: $P_{13,Newton}(k)$
: $P_{22}+P_{13,Newton}(k)$
: $P_{13,Einstein}(k)$
Power spectrum [Mpc/h]^3
wavenumber [h/Mpc]
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0.0001
0.0010
0.0100
0.1000
1.0000

Large-scale limit (k→0, r→∞):

$$
\begin{aligned}
\langle|\delta(k,t)|^2\rangle &= \langle|\delta_1(k,t)|^2\rangle + \frac{1}{98}\frac{k^3}{(2\pi)^2}\int_0^\infty dr\,|\delta_1(kr,t)|^2\int_{-1}^{1} dx\left|\delta_1\left(k\sqrt{1+r^2-2rx},t\right)\right|^2\frac{(3r+7x-10rx^2)^2}{(1+r^2-2rx)^2}\\
&+\frac{1}{252}\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-42r^4+100r^2-158+\frac{12}{r^2}+\frac{3}{r^3}\left(r^2-1\right)^3\left(7r^2+2\right)\ln\left|\frac{1+r}{1-r}\right|\right]\\
&+\frac{10}{21}\left(\frac{\ell}{\ell_H}\right)^2\frac{k^3}{(2\pi)^2}|\delta_1(k,t)|^2\int_0^\infty dr\,|\delta_1(kr,t)|^2\left[-\frac{41}{6}r^2-21-\frac{45}{4}\frac{1}{r^2}\right.\\
&\qquad\left.+\frac{9}{8}\frac{1}{(1+r)^2(1-r)^2}\left(5r^6-13r^4+9r^2+1-\frac{2}{r^2}\right)+\frac{3}{16}\left(43r^2+46r-\frac{53}{r}-\frac{36}{r^3}\right)\ln\left|\frac{1-r}{1+r}\right|\right]\\
&\equiv P_{11}+P_{22}+P_{13,\mathrm{Newton}}+P_{13,\mathrm{Einstein}},
\end{aligned}
$$

$P_{13,Newton}$

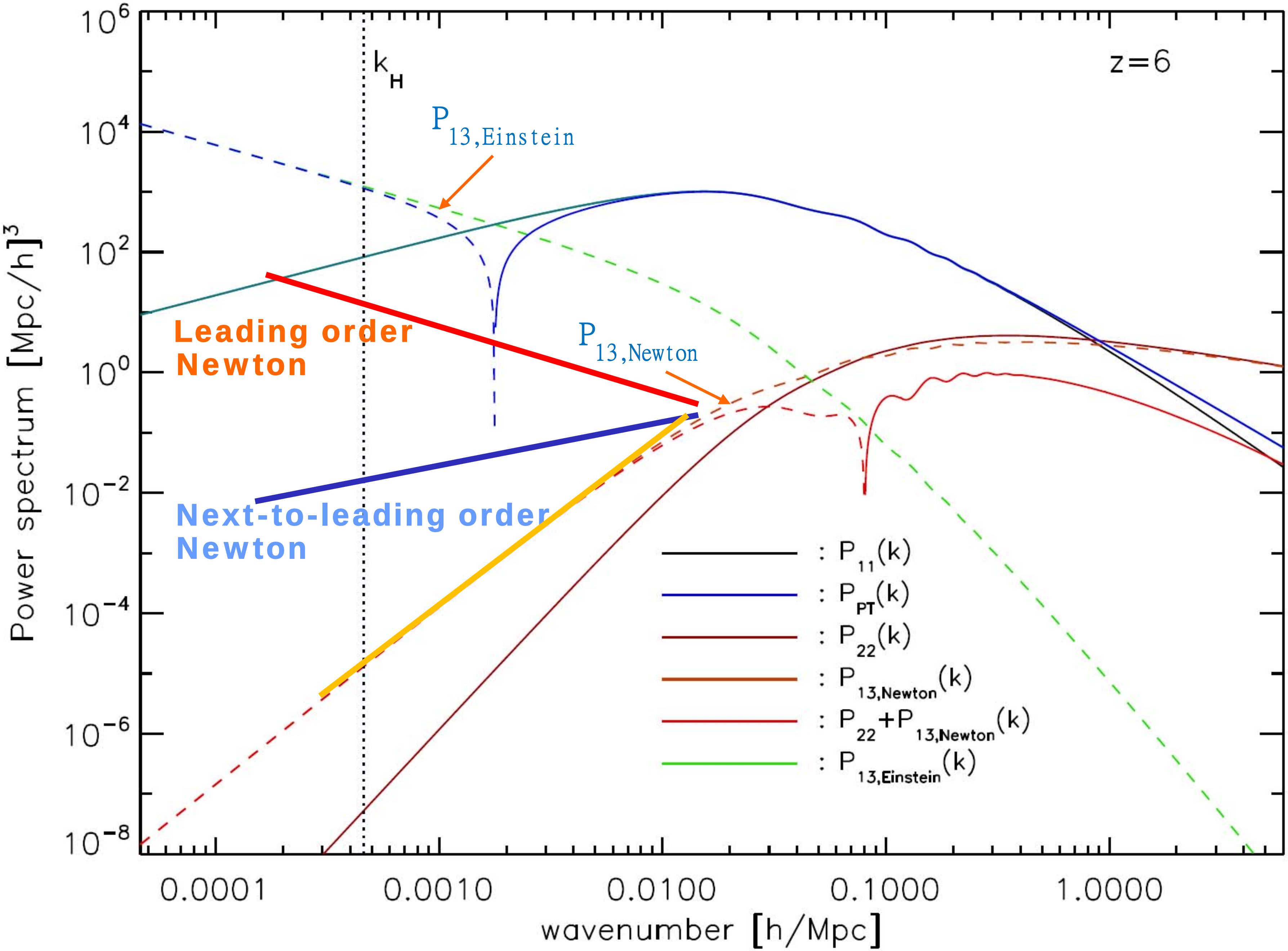
$P_{13,Einstein}$

Leading order
Newton

Next-to-leading order
Newton

In the large scale we discover an infrared divergence in the next-to-leading-order power spectrum due to pure Einstein gravity contribution appearing in the third-order perturbation.

← Despite cancellations of the leading-order and next-to-leading order terms in $P_{13,\mathrm{Newton}}$, no such cancellations occur in $P_{13,\mathrm{Einstein}}$.



z=6
k_H
$P_{13,Einstein}$
$P_{13,Newton}$
Leading order
Newton
Next-to-leading order
Newton
: $P_{11}(k)$
: $P_{PT}(k)$
: $P_{22}(k)$
: $P_{13,Newton}(k)$
: $P_{22}+P_{13,Newton}(k)$
: $P_{13,Einstein}(k)$
Power spectrum [Mpc/h]^3
wavenumber [h/Mpc]
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0.0001
0.0010
0.0100
0.1000
1.0000

Infrared Divergence of Pure Einstein Gravity Contributions to the Cosmological Density Power Spectrum

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We probe the pure Einstein gravity contributions to the second-order density power spectrum. On the small scale, we discover that Einstein's gravity contribution is negligibly small. This guarantees that Newton's gravity is currently sufficient to handle the baryon acoustic oscillation scale. On the large scale, however, we discover that Einstein's gavity contribution to the second-order power spectrum dominates the linear-order power spectrum. Thus, the pure Einstein gravity contribution appearing in the third-order perturbation leads to an infrared divergence in the power spectrum.

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